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GRADIENT METHOD IN THE PROBLEM OF MINIMIZATION OF COLLISION RISK

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Abstract: *The development of surveillance tools and computer systems raises the question of the transition from standard navigation methods to the construction of algorithms for risk field analysis. The field of risks allows to use not only methods of sliding on lines of the set risk, but also gradient procedures. The main advantage of gradient procedures is the preservation of sensitivity to each target anywhere in the field of operations. This property allows you to build algorithms that guarantee the movement of the trajectory of minimal risk, even with the error of the previously laid route. The report presents the mechanisms of formation of risk fields, construction of optimal trajectories and visualization of the situation.*

Keywords: *risk field, gradient procedure, optimal control, sliding along the line of a given risk, mathematical modeling.*

МЕТОД ГРАДІЄНТУ В ЗАДАЧІ МІНІМІЗАЦІЇ РИЗИКУ ЗІТКНЕННЯ

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Анотація: *Розвиток засобів спостереження і обчислювальних систем дозволяє ставити питання про перехід від стандартних методів навігації до побудови алгоритмів аналізу полів ризиків. Поле ризиків дозволяє використовувати не тільки методи ковзання по лініях заданого ризику, а й градієнтні процедури. Головною перевагою градієнтних процедур є збереження чутливості до кожної цілі у будь-якому місці поля операцій. Дана властивість дозволяє будувати алгоритми, що гарантують рух по траєкторії мінімального ризику навіть при помилковості прокладеного раніше маршруту. У доповіді наведені механізми формування полів ризиків, побудови оптимальних траєкторій і візуалізації ситуації.*

Ключові слова: *поле ризиків, градієнтна процедура, оптимальне керування, ковзання по лінії заданого ризику, математичне моделювання.*

When solving the problem of navigation in conditions of risks caused by natural causes and the presence of other vessels, we first determine the criterion of optimality. In the general case, we are dealing with the problem of optimal control. In the works [1–12], the authors previously considered the issues of optimizing the route using the risk criterion to minimizing the risk of collision. Consider the subintegral expression of the integral functional of the goal. First, we consider the expected risks C (x), estimated at the economic, technical and other costs taken into account in the task. In the works [13–18] the authors previously considered one of the main issues that it is not very reasonable to prefer only the risk of collision, because there is also the cost of cargo and other factors. On the other hand, experience says that the risk is determined mainly by various little things. Thus, assuming a certain value of the maximum penalty C_m, we will take the estimate of the current expected risk in the form of the maximum penalty weighted by the normal distribution [19–24]:

$$C(\mathbf{x}) = C_m f(\mathbf{x}) = \frac{C_m}{2\pi\sigma_x\sigma_y} e^{-\frac{1}{2-2r_{xy}}\left[\frac{(v_x t-x_0)^2}{\sigma_x^2} - \frac{r_{xy}(v_x t-x_0)(v_y t-y_0)}{\sigma^2{}_x\sigma^2{}_y} + \frac{(v_y t-y_0)^2}{\sigma_y^2}\right]} . \qquad (1)$$

Thus, we got a risk field. It is important that this field extends to the entire space of the operation and the gradient procedure leads to the point of the vessel's position "in the blind" from any point in the space of operation. The second important point is the impossibility to guarantee complete safety. Therefore, we choose a given level of risk *C*(x)*. The locus of equal risks is the ellipse:

$$\frac{(x-x_0)^2}{a^2} + \frac{(y-y_0)^2}{b^2} = R^2 . \qquad (2)$$

Considering the ellipse of the risk function with the position of the vessel, its direction of movement and characteristics, in the risk field we obtain the line of the given risk. Obviously, any object or vessel moving along a given risk line has been identified and assessed. On the other hand, inward deviation of the equal risk ellipse increases the risk, while outward deviation decreases the risk but increases the path. Thus, we obtain a vector function as an integrand of the goal functional in the form (3):

$$\varphi(\mathbf{x}) = \begin{bmatrix} S \\ C \end{bmatrix}; \qquad L(\mathbf{x}) \to \min \int_L \varphi(\mathbf{x}) \partial \mathbf{x} . \qquad (3)$$

Task (3) itself is a vector problem of optimal control of a distributed system. For all its theoretical complexity [rai], it has a simple solution – you need to move along the line of equal risk, then the distance covered in the maneuver will be minimal for a given risk, and the risk will not exceed a given one. Then, during the maneuver, the trajectory of own ship is determined by the slip equation:

$$\left.\begin{aligned} \frac{(v_T\cos\theta_T - x_{T0})^2}{a^2} + \frac{(v_T\sin\theta_T - y_{T0})^2}{b^2} = R^2 \\ (v_h\cos\theta_h - x_{h0})^2 + (v_h\sin\theta_h - y_{h0})^2 = r \end{aligned}\right\}. \tag{4}$$

To make a decision on the choice of risk-minimizing maneuvers, we calculate the reference risk field, which is the same for all tasks – the normal distribution in Fig. 1.

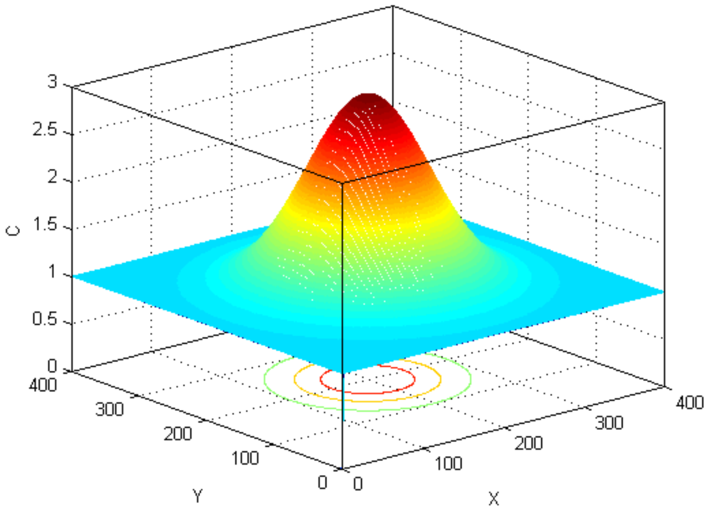


Figure 1 – Reference risk distribution

To obtain an instantaneous risk field associated with a specific vessel, we perform an affine transformation of the reference risk field. For this, we introduce the heading angle of the target φ_c, the standard deviations of the target along the axes σ_x σ_y and the maximum risk C_m. This data allows the construction of an instantaneous risk field for a given target. To do this, we transform the original field, Fig. 2, using transform:

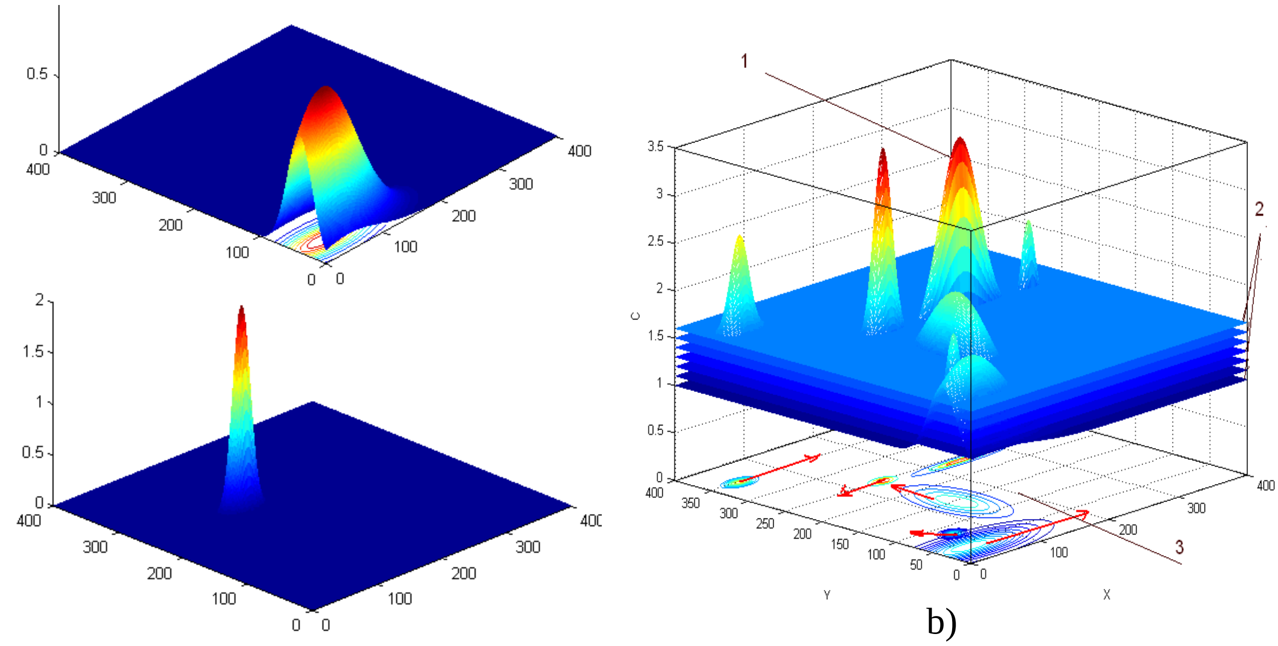
$$\mathbf{x} = C_m A\mathbf{x}_1 + \mathbf{x}_0 \quad \rightarrow \quad \begin{bmatrix} x \\ y \end{bmatrix} = C_m \begin{pmatrix} \frac{\cos\varphi}{\sigma_x} & \sin\varphi \\ \cos\varphi & -\frac{\sin\varphi}{\sigma_y} \end{pmatrix} \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} + \begin{bmatrix} x_0 \\ y_0 \end{bmatrix}. \tag{5}$$

This fast operation of transformation of coordinates allows obtaining risk fields of targets (Fig. 2, a). To obtain the entire instantaneous risk field, the operation of adding the target risk fields is performed (Fig. 2, b).

$$C(\mathbf{x}) = \sum_{i=1}^{n} C_i(\mathbf{x}). \tag{6}$$

The risk field extends to the entire operation field and at each point of the operation field there is a component of each of the private risk fields and at each point of the risk field you can determine the gradient of this field. Therefore, when moving a vessel, you can always choose a direction that does not lead to an area of increased risk. Thus, there is a procedure that will always lead to entering the area of unacceptable risks of the target is a gradient procedure

$$\begin{aligned} &\mathbf{x}_{i+1} = \mathbf{x}_i + \frac{v}{|gradC(\mathbf{x})|} gradC(\mathbf{x}); \quad i = 1,\dots,m; \\ &\mathbf{x}(0) = \mathbf{x}_0 \end{aligned} \tag{7}$$



a) b)

Figure 2 – Formation of the risk field of the task: a) the risk fields of targets; b) the result of adding up the risk fields of targets

The most "dangerous" trajectory is the condition of sufficient self-speed. The existence of the most "dangerous" trajectory implies the presence of the most "safe" trajectory, where a collision is fundamentally impossible, again, provided there is sufficient speed to perform the evasive maneuver. This is the sliding trajectory in the direction of the orthogonal allowable risk gradient

$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \mathbf{x}_{i+1} = \mathbf{x}_i + \frac{v}{|gradC(\mathbf{x})|} gradC(\mathbf{x}) \rightarrow \mathbf{x}_{i+1} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \mathbf{x}_{i+1};$$
$$\mathbf{x}(0) = \mathbf{x}_0 \tag{8}$$

Now each step on a trajectory is followed by sliding on a line of an equal gradient and it is important only to control a condition of not exceeding the set risk

$$C(\mathbf{x}) <= C^*;$$
$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \mathbf{x}_{i+1} = \mathbf{x}_i + \frac{v}{|gradC(\mathbf{x})|} gradC(\mathbf{x}) \rightarrow \mathbf{x}_{i+1} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \mathbf{x}_{i+1}; \tag{9}$$
$$\mathbf{x}(0) = \mathbf{x}_0$$

In case of divergence with several goals, it is necessary to keep a safe distance from all goals. This complements the algorithm by requiring the choice of the beginning of the path on the minimum gradient, in the future if enough of its own speed, the ship will automatically slide between targets on a safe trajectory. It is necessary to have the starting point of the maneuver on the trajectory of the maneuver 1, and the end point of the maneuver 1* belonging to the trajectory of the general course L*. Thus, we obtain a simple algorithm for "blind" maneuvering under the control of an automatic system that has constantly updated information about the situation in the area of the operation and seeks to avoid increased risk

$$\mathbf{x}(0) = \mathbf{x}_0 \in L^*$$
$$\mathbf{x}_0 \to \min gradC(\mathbf{x})$$
$$C(\mathbf{x}) <= C^*; \qquad (10)$$
$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\mathbf{x}_{i+1} = \mathbf{x}_i + \frac{v}{|gradC(\mathbf{x})|} gradC(\mathbf{x}) \to \mathbf{x}_{i+1} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\mathbf{x}_{i+1}.$$
$$\mathbf{x}_m \in L^*$$

A simplified algorithm of the maneuvering procedure using the risk field gradient is shown in Fig. 3.

The basis of the algorithm (Fig. 3) is the analysis of the gradients of the risk field.

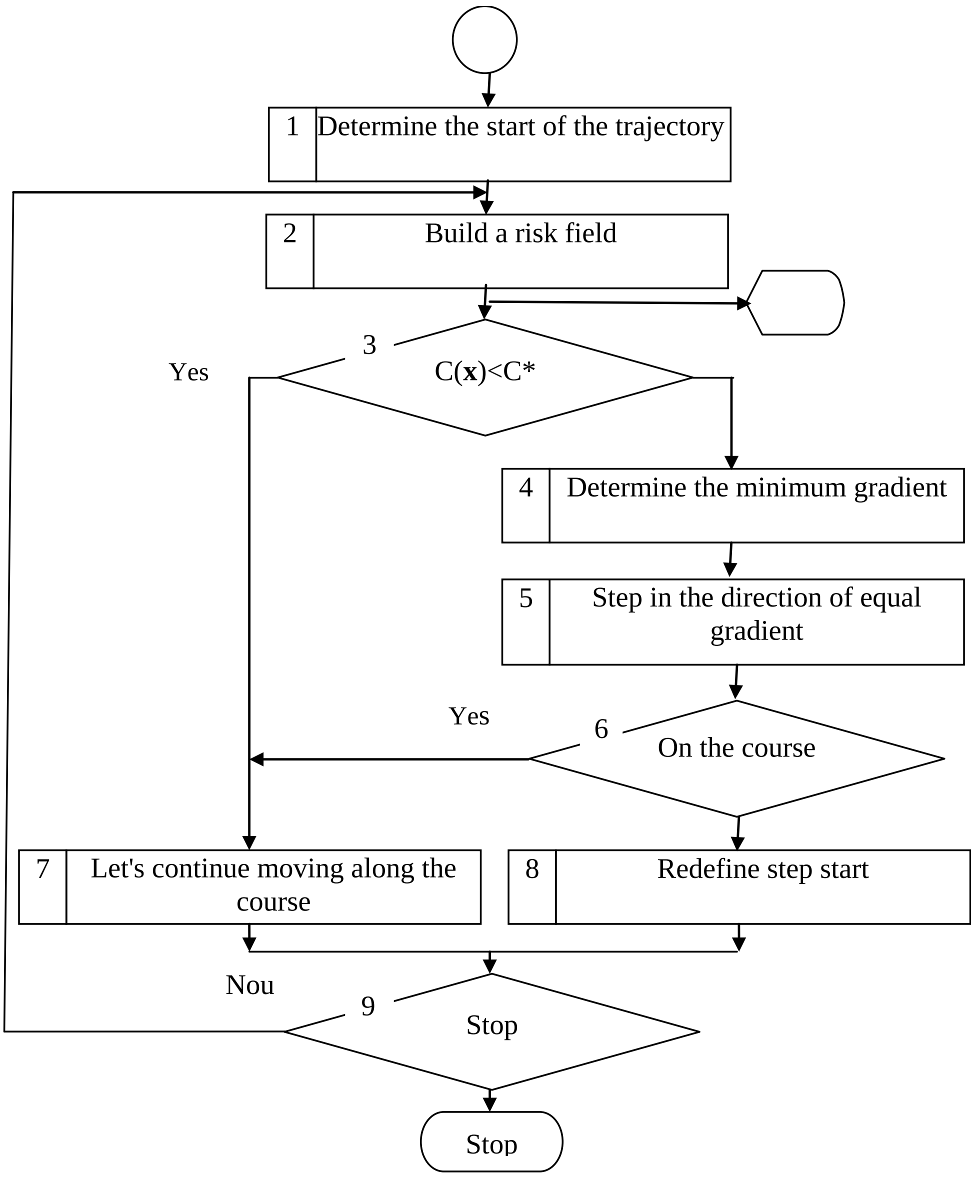


Figure 3 – Algorithm of maneuvering using the gradient of the risk field

Conclusions. The use of the risk field in the navigation task allows us to construct gradient procedures for laying a course with the least risk.

When constructing the risk field of the target, it is enough to know only the coordinates, own risk, parameters of the distribution of the risk function, which makes it possible to use the extension of the ship's transponder code for identification.

The gradient procedure makes it possible to construct an algorithm for the automatic maneuvering of a vessel while maintaining the minimum risks.

The procedure for sliding along the line of a given risk and the procedure for gradient trajectory construction are compatible and complementary to each other.

The considered algorithms make it possible to form a natural and easily perceived interface of the situation analysis system for the navigator.

Both the sliding algorithm along the line of a given level and the gradient algorithm require accurate knowledge of the dynamic properties of the vessel, which requires in-line identification of the vessel model.

The considered algorithms are intended for the construction of automatic navigation systems and require the use of modern computing systems capable of processing data fields, which is associated with the need to perform simple operations on high-dimensional matrices.

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