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AUTOMATIC PLANNING OF THE SHIP'S ROUTE IN THE RISK FIELD USING GRADIENT PROCEDURES AND A GIVEN COLLISION RISK

Abstract. One of the points of the main document defining the national transport strategy of Ukraine as a maritime state for the period until 2030, approved by the order of the Cabinet of Ministers of Ukraine dated May 30, 2018 No. 430-r, is the implementation of intelligent transport systems and traffic control systems on land and water transport [1].

The main goal of the article is the analysis and development of methods for improving shipping safety, ensuring the economic indicators of maritime transport through the use of automated systems for preparing a ship for a voyage with automatic modules for planning the ship's route, controlling the movement of the ship while navigate along the route using gradient procedures, which would allow optimizing the joint task of routing, increased the efficiency of automated systems and significantly reduced the influence of the human factor on control processes. Based on the analysis of international publications, regarding the analysis of marine transport accidents, it is stated that the human factor is a determining obstacle for ensuring the safety of navigation and optimizing the processes of managing the movement of ships. In conclusion, the use of automated control systems based on the use of modern algorithms for optimal planning and execution of operations is proposed to increase the safety of navigation, improve the economic performance of voyage tasks and reduce the influence of the human factor.

The work deals with the issue of automatic routing of the ship's route in the field of risks. The algorithm of automatic routing of the ship's route is given and a description of the main functional blocks is given. It is shown that the use of the field of risks allows maintaining convenient screen forms for visualization of the field of maneuvering operations when moving along the route. The issue of the use of gradient methods in the task of laying the ship's route is considered. It is noted that: the smoothness of risk fields allows the use of simple gradient procedures for solving optimization problems. The conditions for using gradient procedures in the tasks of laying a route and performing a divergence maneuver when a ship is moving along a route with sea objects and obstacles are given. A method of planning the route of the ship's movement has been developed, which consists in constructing a risk field in the on-board computer using gradient procedures and a given risk of collision, unlike the existing ones, it provides automatic route planning and prompt display of data, which allows you to automate routine operations and reduce the time of laying the route of the ship's movement, to reduce the distance of the ship along the route and reduce fuel consumption. Mathematical modeling of the processes of laying the ship's route was carried out using the developed procedures and gradient methods.

Keywords: intelligent transport system, navigation safety, automated system, human factor, risk field, on-board computer, gradient method, mathematical modeling.

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АВТОМАТИЧНЕ ПЛАНУВАННЯ МАРШРУТУ РУХУ СУДНА У ПОЛІ РИЗИКУ ІЗ ВИКОРИСТАННЯМ ГРАДІЄНТНИХ ПРОЦЕДУР ТА ЗАДАНОГО РИЗИКУ ЗІТКНЕННЯ

Анотація. Одним із пунктів основного документу, що визначає національну транспортну стратегію України як морської держави, на період до 2030 року, схваленого розпорядженням Кабінету Міністрів України від 30 травня 2018 р. № 430-р, є впровадження інтелектуальних транспортних систем та систем керування рухом на наземному та водному транспорті [1].

Головною метою статті є аналіз та розробка методів щодо підвищення безпеки судноплавства, забезпечення економічних показників морського транспорту шляхом використання автоматизованих систем підготовки судна до рейсу з автоматичними модулями планування маршруту судна, керування рухом судна при русі по маршруту з використанням градієнтних процедур, що дозволяло б оптимізувати спільну задачу прокладання маршруту, підвищило ефективність автоматизованих систем та суттєво зменшило вплив людського чиннику на процеси керування. На підставі аналізу міжнародних видань, щодо аналізу аврійності на морському транспорті, зазначено, що людський чинник є визначальною перешкодою для забезпечення безпеки судноводіння і оптимізації процесів керування рухом суден. У підсумку, запропоновано використання автоматизованих систем управління, які базуються на використанні сучасних алгоритмів оптимального планування та виконання операцій, для підвищення безпеки судноводіння, покращення економічних показників виконання рейсового завдання та зменшення впливу людського чинника.

У роботі розглядаються питання автоматичної прокладки маршруту руху судна у полі ризиків. Наведено алгоритм автоматичної прокладки маршруту руху судна та дано опис основних функціональних блоків. Показано, що використання поля ризиків дозволяє підтримувати зручні екранні форми для візуалізації поля операцій маневрування при русі по маршруту. Розглянуті питання використання градієнтних методів у задачі прокладки маршруту руху судна. Зазначено, що: гладкість полів ризиків дозволяє використовувати прості градієнтні процедури для вирішення оптимізаційних задач. Приведені умови використання градієнтних процедур у задачах прокладки маршруту і виконанні маневру розходження при русі судна по маршруту із морськими об'єктами та перешкодами. Розроблено метод планування маршруту руху судна, який полягає у побудові у бортовому обчислювачі поля ризиків із використанням градієнтних процедур та заданого ризику зіткнення, на відміну від існуючих забезпечує автоматичне планування маршруту та оперативне відображення даних, що дозволяє автоматизувати рутині операції та скоротити час прокладання маршруту руху судна, скоротити дистанцію руху судна по маршруту та зменшити витрати палива. Проведено математичне моделювання процесів прокладки маршруту руху судна із використанням розроблених процедур та градієнтних методів.

Ключові слова: інтелектуальна транспортна система, безпека судноводіння, автоматизована система, людський чинник, поле ризику, бортовий обчислювач, градієнтний метод, математичне моделювання.

Problem Statement. Controlling the navigation of the ship requires accurate and efficient route planning, which will ensure safety, save fuel and minimize the time required to complete the voyage task. The route planning process must take into account factors such as geographic restrictions, weather conditions, seaway conditions, the presence of obstacles and the need to perform certain maneuvers. To lay out the optimal route of the ship from the port of departure to the port of destination, optimization of the selected optimality criterion is required, which can be implemented only if numerical optimization methods are used in the on-board computer. Such a criterion can be, for example, the integral risk on the ship's route and directly during the maneuver of divergence with sea objects, and the task of optimal planning is the minimization of this risk [2-3].

Analysis of Recent Research and Publications. The existing methods and models of ship route planning, control of ship movement along the route, and the maneuver of separation with sea objects are based on the concept of a safe area, which is built around each ship and moves with the ship in the direction of the ship's movement, and is called the "ship domain". For a long time, this area was taken as a circle, then, based on the results of statistical studies of the movement of ships in the inland waters of Japan, Japanese scientists Fujii and Tanaka proposed a "ship domain" in the form of an ellipse, which is more natural, since the ship has an elongated shape [4-6]. The security concept, built using the concept of the ship domain, has a significant drawback - it copies the manual routing of the route. More promising is the transition to the ship domain, which is described by the law of probability distribution (for technical systems by the normal law of Gaussian distribution). The task of optimal course routing first of all requires the determination of the criterion of optimality or the objective function. The collision risk can be chosen as such a function $C(\mathbf{x})$, and the solution to the optimization problem consists in minimizing this risk $C(\mathbf{x})$ on the ship's trajectory. Weather conditions and other obstructions that make navigation difficult are combined into equality-type restrictions $\varphi_i(\mathbf{x}) = 0$, $i = 1,...,m_1$ and unevenness-type restrictions $\varphi_i(\mathbf{x}) < 0$, $i = 1,...,m_2$. In the simple case of navigation, the problem takes the form of a Lagrange task [7-9]. The solution of this optimization problem of laying the route and the divergence of ships when sailing along the route is performed according to the criterion of minimum consumption under the condition of optimality [10-13].

$$\frac{\partial C_i}{\partial C_j} = 0; \quad i = 1..m, j = 1..m, i \neq j;$$
$$\frac{\partial C_i}{\partial C_i} = 1; \quad i = 1..m. \qquad (1)$$

Purpose of the Article. The purpose of the article is to research the effectiveness of automated ship traffic management systems in conditions of navigational risks: automation of route planning processes, optimization of divergence maneuvers, taking into account the interests of all participants in the operation during route planning and divergence, reducing the influence of the human factor on ship traffic management processes, increasing efficiency and safety of navigation.

Presentation of the main research material. The navigational passage of the ship from the port of departure to the port of destination can be carried out by various routes. The choice of a safe route for a vessel is affected by navigation restrictions, meteorological conditions for sailing at different times of the year, crossing IMO-approved pirate zones, changes in the direction and speed of currents, the presence of other vessels on the route, etc. To lay out the optimal route of the ship, optimization of the selected optimality criterion is required, which can be implemented only if numerical optimization methods are used in the on-board computer. Such a criterion can be, for example, the integral risk on the ship's route and directly during the execution of the maneuver of divergence with target-ships, and the task of optimal planning is the minimization of this risk

$$L^{*}(\mathbf{x}(t)) \to \min C;$$

$$C(\mathbf{x}) = \frac{C_m}{2\pi\sigma_x\sigma_y}\int\limits_{L(t)} e^{-\frac{1}{2-2r_{xy}}\left[\frac{(v_x t - x_0)^2}{\sigma_x{}^2} - \frac{r_{xy}(v_x t - x_0)(v_y t - y_0)}{\sigma^2{}_x\sigma^2{}_y} + \frac{(v_y t - y_0)^2}{\sigma_y{}^2}\right]} dt \qquad (2)$$

where: σ_χ ,σ_γ - total root mean square errors of the above-mentioned uncertainties along the axes of the ship-related coordinate system;

r_{xy}- the correlation coefficient of the total root mean square errors;

C_m- the ratio of the value of the ship and cargo in the distribution of the total risk;

x_0, y_0 - coordinates of the mathematical expectation of the distribution $M\left\{C(\mathbf{x})\right\}$;

x, y - coordinates of an arbitrary point in the field of operations.

In fig. 1 shows the main stages of automatic routing of the ship, taking into account the given risk

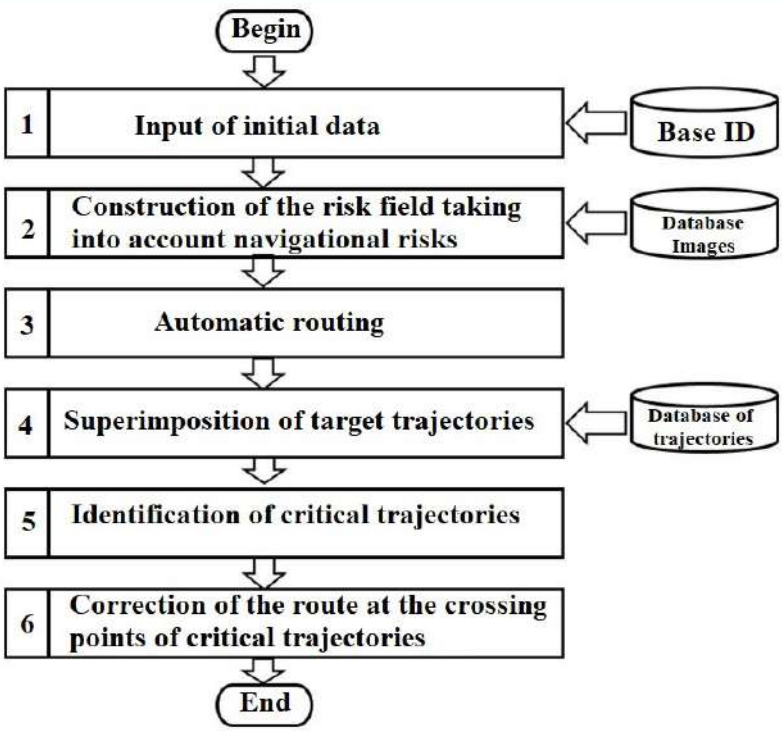


Fig. 1. *The main stages of automatic routing of the ship, taking into account the function of the level of danger*

Block 1 "Initial data entry" is intended for automatic entry from the database of all initial conditions (coordinates of the initial point "departure port", coordinates of the final point "arrival port", initial course, navigation area, rms data of uncertainties, values of the given risk, navigation, hydrographic and weather restrictions, other data in electronic form used in route planning.

Block 2 "Construction of the risk field taking into account navigational risks" is intended for: construction of the risk field of the swimming area based on satellite images or electronic map information of the swimming area; construction of contours of equal risks and selection of the contour of a given risk; display of risk contours on the terminal.

Block 3 "Automatic route planning of the ship's route" is intended for automatic planning of the optimal route of the ship's movement in the constructed field of risks using gradient methods.

In fig. 2 shows a simplified procedure for automatically plotting the ship's route taking into account the risk level function.

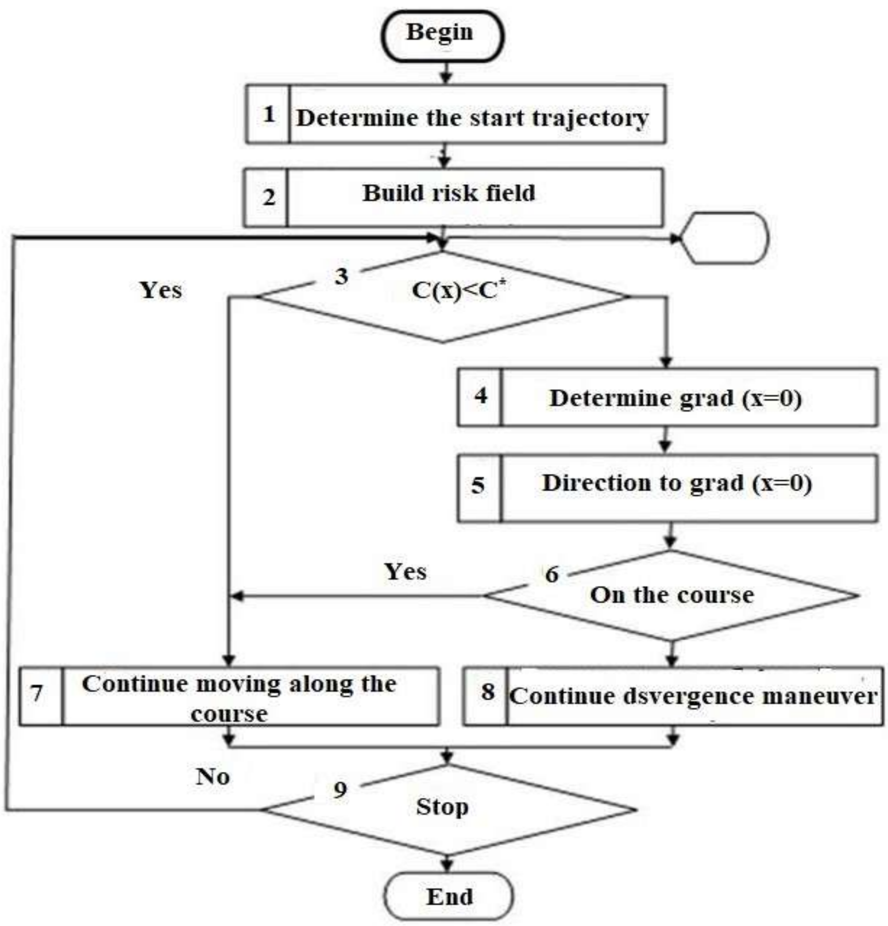


Fig. 2. *A simplified algorithm for automatically plotting the ship's route taking into account the risk level function*

From the determined starting point of the ship's route (Fig. 2), a step is taken in the direction of the given course and the risk at the new point of the route is assessed. If the risk value at the new point does not exceed the given level (estimated by logic block 3), then we continue sailing in the direction of the shoulder of the ship's route. If the risk at the new point exceeds the given one, the zero gradient of the risk field at the current point is calculated and a step is taken in the direction of this gradient. This is repeated until the trajectory of bypassing the navigation risk does not cross the shoulder of the ship's movement route, on which this navigation risk is located, after which the algorithm takes steps in the direction of the given shoulder of the ship's route.

In fig. 3 shows the scheme of automatically ploting the ship's route when avoiding a navigational risks

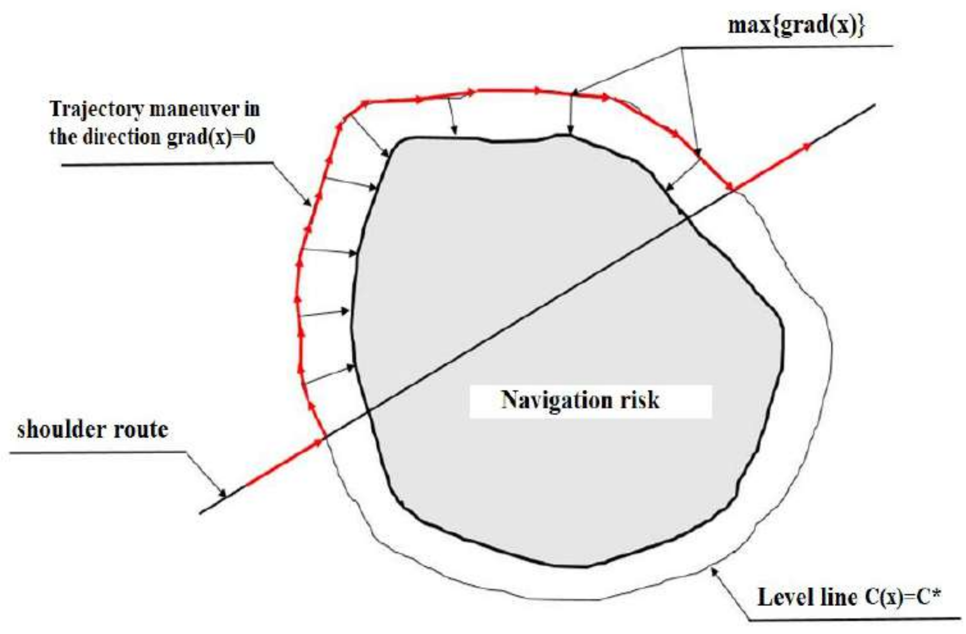
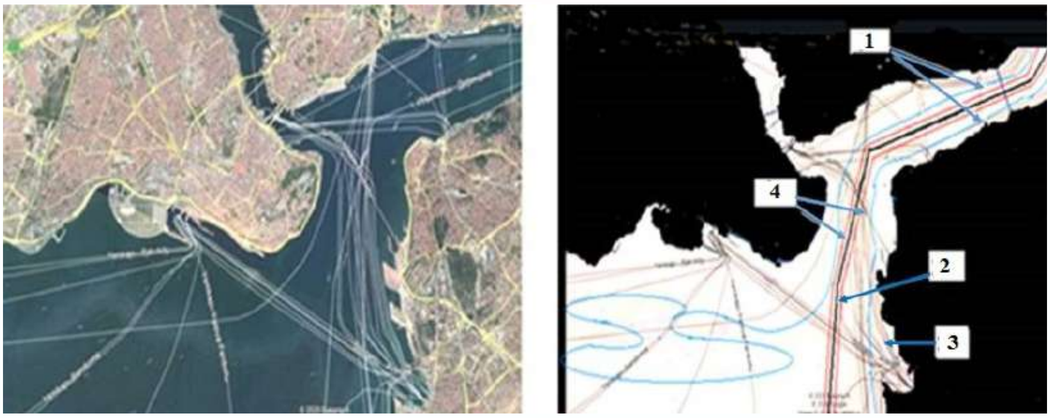


Fig. 3. *Scheme of automatic ploting of a route when avoiding a navigational risks*

Block 4 "Superimposition of trajectories of target-vessels" is intended for superimposing nominal trajectories of all target-vessels from the database of target- vessel trajectories of a given navigation area on the constructed risk field and displaying them on the monitor.

In fig. 4.a),b) shows a satellite image of the sailing area with the trajectories of the target-vessels and the trajectories of the own vessel and a digital image of the sailing area for further processing in the on-board controller.



а) б)

Fig. 4. *Satellite image of the navigation area and its digital processing*

The blue lines, position 1, define the critical risk caused by shallow water near the shoreline. The black line, position 2, determines the trajectory of sailing and the maximum value of the risk field of one's own vessel. The purple lines, position 3, define the routes and maximum risks created by the target-vessels. The red lines, position 4, define the corridor of dangerous risks of one's own vessel.

Block 5 "Detection of critical trajectories of movement" is designed to identify critical trajectories of vessels of targets with which a collision is possible, Fig. 5.

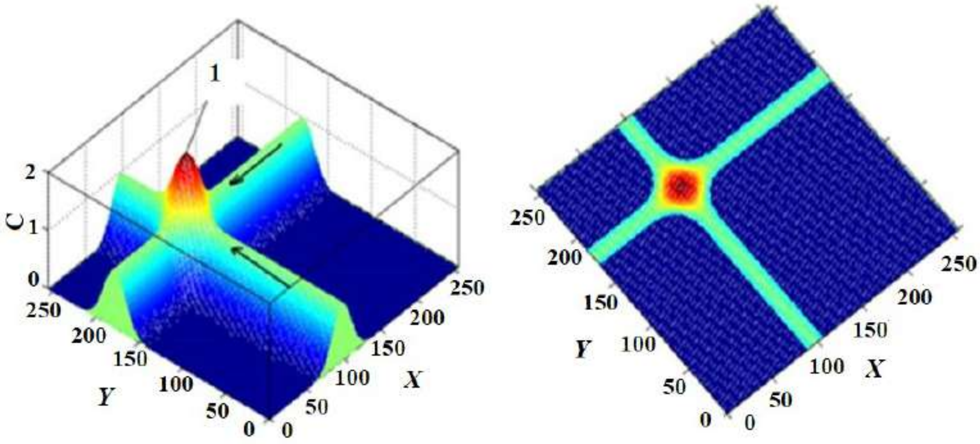


Fig. 5. *Critical trajectory of movement in relation to a possible collision*

Collision point 1 can be active if the own ship and the target-ship arrive at it at the same time, or passive if the own ship and the target-ship arrive at it at different times with an interval that does not exceed the given risk $C_к$.

$$t \in [t_0, T]; \qquad \varepsilon_k = \begin{cases} 1 & if \left\| \mathbf{x}_1(T) - \mathbf{x}_2(T) \right\| \geq C_k \\ 0 & if \left\| \mathbf{x}_1(T) - \mathbf{x}_2(T) \right\| < C_k \end{cases} \qquad (3)$$

Trajectories of the ship's motion containing active collision points are called critical. For non-critical trajectories of movement, it is impractical to remove their risk fields, since a certain risk remains even for a guaranteed non-critical trajectory of movement. This is due to the "human factor", observation error, unexpected maneuver of the target-vessel.

In fig. 6. a fragment of the navigation area with an image of the critical trajectory of the vessel's navigation is given

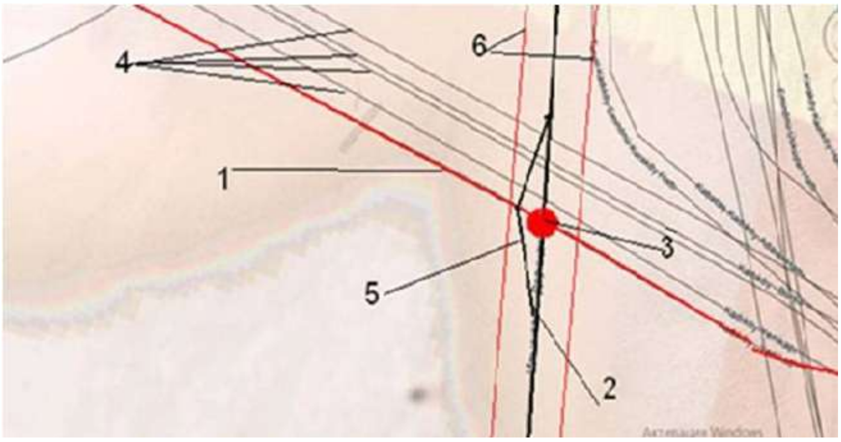


Fig.6. *A fragment of the navigation area with an image of the critical and non-critical trajectories of the ship's movement*

Critical trajectory of movement 1, trajectory of movement of one's own vessel 2, collision points 3, non-critical trajectories of movement 4, trajectory of movement when performing a maneuver to avoid collision with critical trajectory of movement of the target-vessel 5, corridor of risks of one's own vessel 6.

To determine the criticality of the vessel's trajectory, block 5 calculates the position of the target-vessel moving along this trajectory at the time of its own vessel's location at point 3.

If the distance between the ship, at the moment of its location in point 3, and the target is greater than the given risk zone, then the trajectory is not critical. Otherwise, the trajectory of the target vessel is critical and there is a need to adjust the trajectory of one's own vessel for safe divergence from the target-vessel.

Block 6 "Correction of the route at the crossing points of the critical trajectories of the ship's movement" performs the correction of the trajectory of the own ship in the vicinity of the collision point 3 within the field of permissible risks, position 6. In fact, in this case, the task of optimal divergence of the trajectory of a given risk is solved, which will be considered in next section.

In fig. 7. an illustration of the process of correction the route at the point of intersection with the critical trajectory using gradient procedures is given.

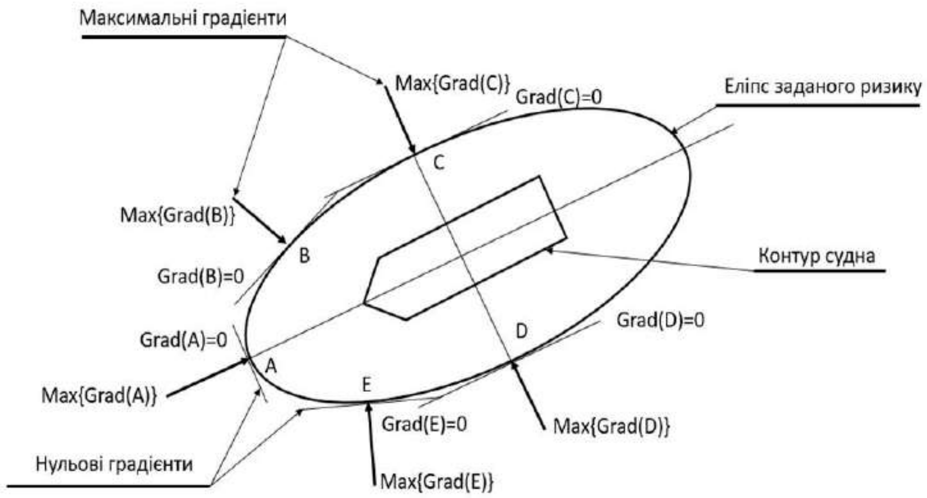


***Fig.7.** Illustration of the process of adjusting the route at the point of intersection with the critical trajectory using gradient procedures*

The contour of the ship is graphically depicted - goals, an ellipse of a given risk, maximum gradients at arbitrary points A, B, C, D, E of the ellipse, perpendicular to the contour of the ellipse at these points, and zero gradients at these same points, tangent to the contour of the ellipse. The operation of the algorithm for adjusting the ship's route at the crossing points of critical trajectories is similar to the operation of the algorithm for automatically laying the ship's route when bypassing a navigational hazard (Fig. 3), except that the bypass is performed along the given risk ellipse using gradient procedures.

In the task of planning the ship's route, the field of risks is considered as a set of total risks of many factors: navigational risks $C_{н}(\mathbf{x})$, own risk $C_0(\mathbf{x})$, risks of other target- vessels $C_i(\mathbf{x})$ participating in the operation and others. It is noted that the total risk of each of the factors has a normal loss probability distribution law and can be represented as

$$C_i(\mathbf{x}) = \frac{C_m}{2\pi\sigma_x\sigma_y} e^{-\frac{1}{2-2r_{xy}}\left[\frac{(x-x_0)^2}{\sigma_x^2} - \frac{r_{xy}(x-x_0)(y-y_0)}{\sigma_x^2\sigma_y^2} + \frac{(y-y_0)^2}{\sigma_y^2}\right]}, i = 0..n \qquad (4)$$

Total risks $C_i(\mathbf{x}),\ i = 0..n$ *and the entire field of risks, as the sum of risks* $C_i(\mathbf{x}),\ i = 0..n$, *have a smooth surface, which allows the use of simple gradient* procedures for solving optimization problems. The main advantage of gradient procedures is their simplicity, the ability to significantly reduce the time of searching for an optimal solution and increase the reliability of obtaining the final result.

Let's find the gradient of the total risk $C_i(\mathbf{x})$ *(4) for each of the factors* i $0..n$

$$\begin{cases} \mathbf{grad}C_i(\mathbf{x}) = (grad_x C_i(\mathbf{x}), grad_y C_i(\mathbf{x})) \\ grad_x C_i(\mathbf{x}) = \dfrac{\partial C_i(\mathbf{x})}{\partial x} = -\dfrac{1}{(2-2r_{xy})}\left[\dfrac{2(x-x_0)}{\sigma_x^2} - \dfrac{r_{xy}(y-y_0)}{\sigma_x^2\sigma_y^2}\right]C_i(\mathbf{x}) \\ grad_y C_i(\mathbf{x}) = \dfrac{\partial C_i(\mathbf{x})}{\partial y} = -\dfrac{1}{(2-2r_{xy})}\left[\dfrac{2(y-y_0)}{\sigma_x^2} - \dfrac{r_{xy}(x-x_0)}{\sigma_x^2\sigma_y^2}\right]C_i(\mathbf{x}) \end{cases} \quad (5)$$

since

$$\frac{\partial C(\mathbf{x})}{\partial x_j} = \sum_{i=1}^{n} \frac{\partial C_i(\mathbf{x})}{\partial x_j},$$

$$\frac{\partial C(\mathbf{x})}{\partial y} = \sum_{i=1}^{n} \frac{\partial C_i(\mathbf{x})}{\partial y};$$

then

$$\mathbf{grad}C(\mathbf{x}) = \sum_{i=1}^{n} \mathbf{grad}C_i(\mathbf{x})$$

The gradient of the risk function $\mathbf{grad}C(\mathbf{x})$ *is the most dangerous direction of increasing field risk*

$$\mathbf{x}_{j+1} = \mathbf{x}_j + \frac{v}{|\mathbf{grad}C(\mathbf{x})|}\mathbf{grad}C(\mathbf{x}),\ j = 1..m \qquad (6)$$

$$\mathbf{x}(0) = \mathbf{x}_0$$

where: v *- speed module.*

The presence of the most dangerous direction of risk increase also means the presence of the safest direction $\mathbf{grad}C(\mathbf{x})$ *in the opposite direction and directions with a zero value of the gradient* $\mathbf{grad}C(\mathbf{x}) = 0$, *on which the specified collision risk is preserved, see Fig. 7.*

The directions $\mathbf{grad}C(\mathbf{x}) = 0$ *are perpendicular* $\mathbf{grad}C(\mathbf{x})$

$$\mathbf{x}_{j+1} = \mathbf{x}_j + \frac{v}{|\mathbf{grad}C(\mathbf{x})|}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\mathbf{grad}C(\mathbf{x}),\ j = 1..m \qquad (7)$$

$$\mathbf{x}(0) = \mathbf{x}_0$$

where:

$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ *- clockwise rotation* $\mathbf{grad}C(\mathbf{x})$ *operator on* $\frac{\pi}{2}$

To rotate $\mathbf{grad}C(\mathbf{x})$ *counterclockwise, you need to use the operator* $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$.

In the tasks of route planning or divergence with one target, it is necessary to control not to exceed the given collision risk in addition to (7)

$$\begin{cases} C(\mathbf{x}) \le C^* \\ \mathbf{x}_{j+1} = \mathbf{x}_j + \dfrac{v}{|\mathbf{grad}C(\mathbf{x})|}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\mathbf{grad}C(\mathbf{x}),\ j = 1..m \\ \mathbf{x}(0) = \mathbf{x}_0 \end{cases} \qquad (8)$$

In the tasks of ploting out the route of the vessel's movement or divergence with several target-vessels while moving along the route, it is necessary to ensure that the specified risk of collision is not exceeded for each of them. This imposes an additional requirement to choose a starting point $|\mathbf{grad}C(\mathbf{x}_0)|=0$ in the risk field that corresponds to the smallest risk value $C(\mathbf{X}_0) = \min\{C(\mathbf{X}_0)\}$. Further routing of the ship's movement route or divergence with several target-ships while moving along the route consists in blindly maneuvering along the trajectory L^* according to the system conditions (9)

$$\begin{cases} \mathbf{x}(0) = \mathbf{x}_0 \in L^* \\ \mathbf{x}_0 \to \min|\mathbf{grad}C(\mathbf{x})| \\ C(\mathbf{x}) \le C^* \\ \mathbf{x}_{j+1} = \mathbf{x}_j + \dfrac{v}{|\mathbf{grad}C(\mathbf{x})|}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\mathbf{grad}C(\mathbf{x}),\ j = 1..m \\ \mathbf{x}_m \in L^* \end{cases} \qquad (9)$$

Conclusions. The article deals with the use of the risk criterion in the task of planning the ship's route. It is noted that the traditional concept safety of navigation, built using the concept of the ship domain, has a significant drawback - it copies the manual ploting of the ship's route. It is shown that the transition to the ship domain, which is described by the law of probability distribution, is more promising. Such a transition is based on the fact that risk is always and everywhere and there is no area where there is no risk at all.

The issue of automatic ploting of the ship's route in the field of risks was considered. The algorithm of automatic ploting of the ship's route is given and a description of the main functional blocks is given. It is shown that the use of the field of risks allows maintaining convenient screen forms for visualization of the field of operations.

The issue of the use of gradient methods in the task of ploting the ship's route is considered. It is noted that: the smoothness of risk fields allows the use of simple gradient procedures for solving optimization tasks; the main advantage of gradient procedures is their simplicity, the ability to significantly reduce the time of searching for an optimal solution, and the reliability of obtaining the final result. The conditions for using gradient procedures in the tasks of laying a ship's route and performing a divergence maneuver when moving along a route with marine objects are given.

The actual scientific and practical problem of increasing the efficiency of automated systems for preparing a ship for the execution of a voyage task in terms of laying the route of the ship during the execution of the voyage task and, as a result, reducing the influence of the "human factor" due to the development of scientific and technical foundations, principles and methods of creating automatic modules for laying out the ship's route and departure from sea objects, which ensure the optimization of the functionality of the management quality for all participants of the operation in the field of risks.

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