

## VESSEL DYNAMIC POSITIONING SYSTEM MATHEMATICAL MODEL

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**Annotation.** The work analyses a structure and principles of the dynamic positioning system as a high-tech control complex and also considers the problems of these systems. A mathematical model with the equation of the non-linear motion of the ship with three degrees of freedom for the purpose of obtaining reliable coordinates for keeping the ship at a given point is shown.

The greatest interest is the reaction of the analysed system's to disturbing external influences. The causes of their occurrence are usually wind, current and sea waves. The presence of disturbances entails a deviation of the ship from the desired position, which requires appropriate intervention from the control system. To weaken the specified reaction, various dynamic filters are introduced into the composition of the control law, which suppress the component of the control signal generated by sea waves.

**Keywords.** Dynamic positioning system, offshore vessel, automatic control, simulation, power system.

Currently, maritime transport is an integral part of the global transportation system, so the offshore fleet is developing rapidly.

Depending on their type and type of work they perform, offshore vessels are equipped with a large number of specialized equipment that is not found on ordinary merchant vessels.

The work on offshore vessels is very different from that on merchant vessel due to its specific nature. The task of the merchant fleet is to transport goods between destinations. The offshore fleet consists of special-purpose vessels that perform a variety of specific tasks.

Offshore vessels are special sea vessels used to perform various tasks "offshore".

The activities of offshore vessels include the discovery of oil and gas fields on the sea shelf, drilling oil wells, developing subsea infrastructure, arranging fixed platforms, installing wind turbines, laying transcontinental cables and pipelines, and providing full maintenance of mentioned infrastructure. This is a multitude of construction and installation processes that take place on the high seas. For each type of work, there is a type of vessel of a special design with specialized equipment:

- anchor handling vessel. It specializes in towing drilling rigs, setting and mooring the anchor in the right position;
- cable layers for laying underwater cable networks;

- diving support vessel, one of the most complex in design. Specialization - creation of a floating base for deep-water operations, monitoring, maintenance of pipelines and drilling rigs;

- pipe-laying vessels for pipeline installation;

- drilling vessels. The main mode of operation is drilling of exploration wells.

The equipment includes a system for holding the vessel at the wellheads, racks for storing drill pipes, and tanks for drilling fluids [1].

Electrical equipment and control systems are becoming increasingly complex, requiring high competence of personnel servicing this type of vessels, in particular the support fleet, as well as reliable operation of automatic control systems for vessel components based on correct direct and indirect measurements of current system parameters (rotational speed, pressure, current, voltage, etc.).

Due to their special purpose, offshore vessels have a lot of electrical and electronic equipment, which is maintained by an electrical technical officer [2]. Therefore, obtaining and processing reliable measurement data is the key to the reliable operation of shipboard electrical equipment, and research in this area is relevant.

The task of dynamic positioning for marine surface objects is one of the modern shipping important problems. The number of ships using dynamic positioning systems has been increasing in recent years due to the increased interest in exploration of natural resources in the world's oceans. At the same time, there are many other areas, where dynamic positioning systems are used. These includes support for diving operations, laying underwater pipes and cables, transportation, research tasks, etc.

A dynamic positioning system (DPS) is a system for automatically holding a vessel or offshore structure in a fixed position with or without propeller or thruster.

Dynamic positioning (DP) can be either absolute, meaning that the vessel's position is fixed at a fixed point on the bottom, or relative to a moving object, such as another vessel. It is also possible to position the vessel at a favourable angle to the wind, waves, or current, and this is called a weather vane [3].

The DPS automatically controls the position and course of the vessel with help of constantly operating thrusters and balances the environmental forces (wind, waves, current, etc.).

The environmental forces tend to move the vessel from the desired position, while the automatically controlled thrust balances these forces and keeps the vessel in the desired position.

Dynamic Positioning Systems can be described as the integration of a number of shipboard systems to produce precision manoeuvrability capability.

All of the above suggests are very important when using dynamic positioning systems for fixed positioning of vessels to take into account not only the course position of the vessel itself, but also all the negative forces acting on the vessel. Mathematical modelling of the process is widely used to assess the impact of negative risks.

The ship's position and heading are estimated based on the ship's model, position / heading measurements and the forces acting on the ship itself. Control actions to the engines are calculated based on the difference between the desired position and heading, as well as the approximate position and heading of the vessel. Thrusters

provide the necessary forces to counteract external forces and moments acting on the ship itself.

The measured signals are processed by the signal processing module, this unit performs tests to determine high dispersion, signal spikes and signal drift. False signals are not taken into account and the ship's roll is also compensated.

The main purpose of the estimation module is to provide vessel's position, heading and speed. Fast, purely oscillatory motion caused by pitching has to be filtered out. To estimate the vessel's position, the DPS uses information from a sensor system, i.e., a position measuring equipment or a position reference system of a ship model.

A commonly used feedback controller in the system is a PID-controller that uses position and heading estimates. Some kind of integral action is required to compensate for static environmental disturbances. The controller's feedforward usually consists of a reference feed and a forward feed.

The thruster distribution unit displays the parameters at the controller outputs (such as force and torque) at a given point, such as propeller speed, azimuth, rudder angles, and elevation factor.

The number of papers published in the field of ship modelling is enormous. For example, a nonlinear model in the 6<sup>th</sup> degree of freedom is shown in [5], an overview of ship models and experimental methods for identifying ship dynamics are described in several publications [6, 7].

The hydrodynamic and derivative coefficients encountered in the equations of motion cannot be calculated analytically and, therefore, experimental tests with a physical model can be used to determine these coefficients.

The use of system identification techniques to determine ship's dynamics is increasing and various input signals are being identified. However, the added complexity associated with nonlinear systems, with constraints and lack of initial model structure information, means that an exhaustive search is not always possible. In these cases, an identification strategy can be used to obtain initial values.

In general, a ship model is a set of equations of motion used to predict the motion of a ship when known forces and moments are applied. To achieve good DPS performance, the model should be as detailed as possible. The model parameters must be validated by sea trials. However, the model only represents some of the ship's behavior, so the model is an approximation and is not perfect.

The ship's model includes models of the hull, propulsion and active control devices, such as propeller columns and thrusters, which set the ship in motion and determine its direction.

Like a real object, the ship model is equipped with sensors:

- position and heading sensors to determine the ship's coordinates in space ( $X$  and  $Y$ ) and heading angle;
- motions, which reflect the ship's speed and projections of the speed vector on the  $X$  and  $Y$  axes, as well as the circular speed;
- rotational speed of rudder column propellers (RC) and thrusters (TP), rotation angle of RC and TP;

- external influences: wind - determines the wind speed and direction ( $u, v$ ),  
pitching - determines the pitching score and wave direction.

The external influences model is a simulator of disturbing environmental influences that affect the ship's position. Such influences include waves of different severity and direction and wind acting on the ship with a certain strength and direction [7].

The parameters are read by the relevant sensors of the ship model and are the input data for calculating the forces that cause the ship to move in the longitudinal and transverse directions, the resulting forces are transferred for further accounting and compensation to the calculation blocks of the ship's DPS model.

Without loss of generality, the dynamics of a surface vessel is described by a model based on horizontal motion with variable motion parameters: wave, oscillation, and rotation. The motion in the vessel is determined by hydrodynamic forces and moments, the input variables of which are the shaft angular velocity, which is related to the propeller thrust, and the rudder deflection angle. In this model, it is assumed that roll and pitch changes are not significant and thus can be omitted from the equations. Meaning that the vessel is treated as a solid body moving in a plane, i.e., it has three degrees of freedom:

$$\begin{aligned} m \left( \frac{du}{dt} - vr - x_G r^2 \right) &= X \left( u, v, r, \frac{du}{dt}, \delta, n \right) \\ m \left( \frac{dv}{dt} + ur + x_G \frac{dr}{dt} \right) &= Y \left( v, r, \frac{dv}{dt}, \frac{dr}{dt}, \delta \right) \\ I_z \frac{dr}{dt} + mx_G \left( \frac{dv}{dt} + ur \right) &= N \left( v, r, \frac{dv}{dt}, \frac{dr}{dt}, \delta \right) \end{aligned}$$

where,  $t$  – is the time index,

$u, v$  - wind speed and direction,

$r$  is the angular velocity,

$m$  and  $I_z$  - are the mass of the ship and the moment of inertia relative to the normal axis of the  $X_0Y_0$  plane,

$x_G$  - Cartesian coordinates of the centre of gravity along the  $X_0$  axis,

$\delta$  - is the deviation of the rudder angle,

$n$  - shaft speed,

$X(\dots)$ ,  $Y(\dots)$  and  $N(\dots)$  - external forces (longitudinal waves  $X_0$ , oscillation axis  $Y_0$ ) and moment (for  $X_0$ - $Y_0$  rotation).

Accurate, reliable and continuous location information is essential for dynamic positioning. And DPS requires data at a rate of once per second to achieve high accuracy. The dynamic position of a ship has a certain position reference system independent of the conventional navigation system.

The model implements the DPS mode, which provides automatic control of the vessel's movement from the current position to the specified positioning point by calculating the necessary parameters of active control means. The algorithms incorporated in the DPS model allow to dynamically determine the value of the

emphasis that must be provided by the RPU and PP to compensate for all counteracting forces and achieve the positioning point with a given course.

The DPS model includes calculation modules for distance and heading correction, speed correction, torque correction, accounting for compensated forces, calculating the optimal distribution of thrusts, converting thrusts to angles and propeller speeds.

Measurements from a positioning system cause negative noise. This noise depends on the sensors and the method used to measure the vessel's position. The problem then arises, which is how to estimate the position of the vessel with an approximate knowledge of the vessel's dynamics and with noisy measurements. The answer to this problem is the use of Kalman filtering in engineering. In a dynamic positioning program, the Kalman filter is used to estimate the state of a vessel (for which a dynamic model has been developed) based on noise measurements from the reference system and sensors.

In 1960, R.E. Kalman published his famous article in which he described a recursive solution to the problem of linear filtering of discrete data [8]. Since then, largely due to advances in digital computing, the Kalman filter has been the subject of extensive research and application, particularly in the field of autonomous or assisted navigation.

The Kalman filter estimates a process using a form of feedback control: the filter estimates the state of the process at some time and then receives feedback in the form of (noisy) measurements.

Thus, the equations for the Kalman filter are divided into two groups: time update equations and measurement update equations. The time update equations are responsible for projecting forward (in time) the current state and error covariance estimates to obtain a priori estimates for the next time step. Measurement update equations are responsible for feedback - i.e., for incorporating a new measurement into the a priori estimate to obtain an improved a posteriori estimate.

The time update equations can also be viewed as prediction equations, while the measurement equations can be viewed as correction equations. Indeed, the final estimate of the algorithm resembles a predictor-corrector algorithm for solving numerical problems.

Specific equations for time and measurement updates:

Time update:

$$\begin{aligned}\hat{x}_k &= A\hat{x}_{k-1} + Bu_{k-1} \\ P_k^- &= AP_{k-1}A^T + Q\end{aligned}$$

Updated measurements

$$\begin{aligned}K_k &= P_k^- H^T (HP_k^- H^T + R)^{-1} \\ \hat{x}_k &= \hat{x}_k^- + K_k(z_k - H\hat{x}_k^-) \\ P_k &= (I - K_k H)P_k^-\end{aligned}$$

After each pair of time and measurement updates, the process is repeated and compared to the previous one, the a posteriori estimates are used to design or predict new a priori estimates.

This recursive nature is one of the very attractive features of the Kalman filter - it makes practical implementations much more feasible than (for example) implementing a Wiener filter, which is designed to work with all the data directly for each estimate. Instead, the Kalman filter recursively conditions the current estimate on all past measurements.

The main goal of a positioning control system is to ensure that the ship maintains a given position, regardless of disturbances, and a compass heading that is not affected by disturbance effects. The challenge is to mitigate these disturbances by applying appropriate counteracting forces.

Increased computing power has made it possible to implement more complex control algorithms. More demanding control strategies such as predictive model control and online numerical optimization methods have been commercialized.

Various controllers have been proposed in the literature [9-13]. Some of these controllers have been successfully installed on several commercial DP systems.

Many DPSs rely on multivariate PID-algorithms combined with an observer [14]. The basic principle of the PID-control law is to create a thrust for which different terms are respectively proportional to the 3-dimensional position and deviation vector relative to the ship's position and relative to the desired set point (proportional term), to the speed deviation vector (differential term), and to the accumulated deviation vector (integral term). All of these vectors are related to certain moments of time  $t$ . Based on this principle, the required motor force vector in the housing is fixed and can be formulated as the sum of three terms responsible for proportional, derivative, and integral actions.

The approach to designing new systems provides a step forward in the technology of the ship's dynamic positioning system, which is no less important than the introduction of Kalman filtering in optimal control schemes. The advantages of the latter were so obvious that as a result, the Kalman filtering system was created and is widely used in practice.

**Conclusions.** As a result of a comprehensive analysis of maritime safety problem in different sea surface conditions, it can be concluded that the use of a mathematical model provides a large number of computational studies of ship control modes to create and debug algorithms for controlling ship motion during its positioning, taking into account the optimal use of the resources of the electric power system, which reduces the complexity of developing and debugging control algorithms.

The Kalman filtering method is considered as one of the modern methods of geolocation data filtering. It is advisable to filter first at the data source and then at the information consumer. Preliminary filtering allows you to get rid of redundant and erroneous data, thereby reducing the load on the data transmission channel. The final filtering should be performed on high-performance systems in order to obtain the most efficient filtering.

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